



Year 12 Semester 1 Examination, 2016

Question/Answer Booklet

Hale School

# MATHEMATICS SPECIALIST

Section Two  
Calculator Assumed

<hr/> <i>Student Name</i>
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Teacher:      Mr Hill      Mr Lau  
(circle)

Score:            (out of 99)

### Time allowed for this section

Reading time before commencing work: ten minutes

Working time for this section:                      one hundred minutes

### Materials required/recommended for this section

#### *To be provided by the supervisor*

This Question/Answer Booklet

Formula Sheet (retained from Section One)

#### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,  
and up to three calculators approved for use in the WACE examinations

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	53	35
Section Two: Calculator-assumed	12	12	100	99	65
<b>Total</b>				152	100

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed

(99 Marks)

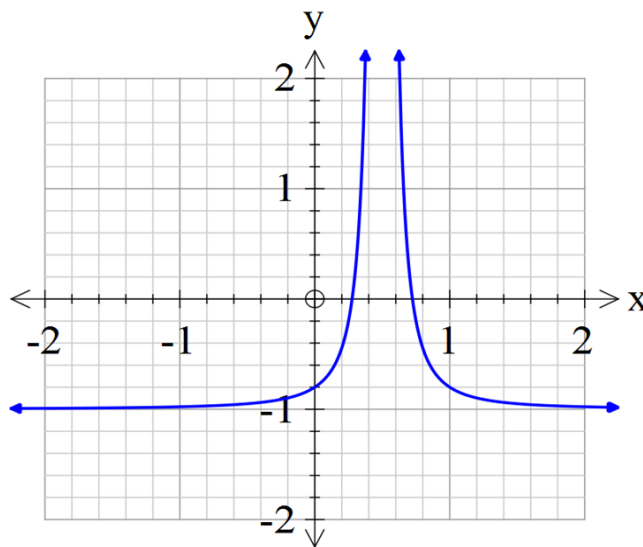
This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

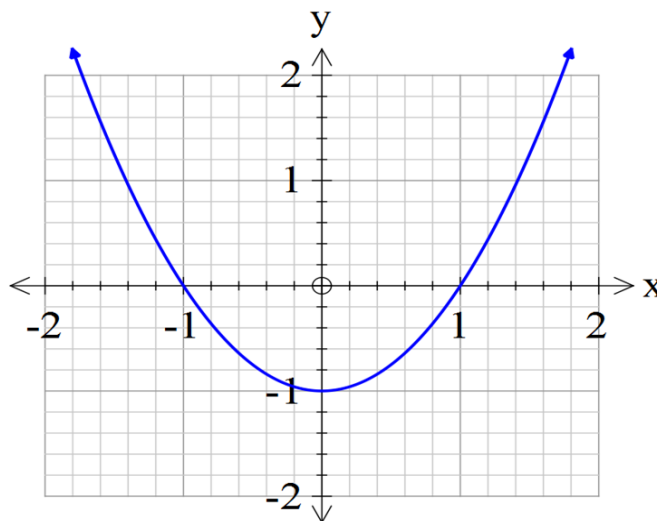
Question 8

(5 marks)

- (a) Given the graph of  $y = f(x)$  below, sketch on the same axes the graph of  $y = |-f(x)|$  (2 marks)



- (b) Given the graph of  $y = f(x)$  below, sketch on the same axes the graph of  $y = \frac{1}{f(x)}$ . (3 marks)



Question 9

(8 marks)

The line  $l_1$  has the equation  $r = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ .

- (a) Find the acute angle between  $l_1$  and the line joining the points  $P \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$  and  $Q \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$ ,  
giving your answer correct to the nearest degree. (3 marks)

- (b) Determine the position vector of the point R that lies on the line joining  $P \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$  and  $Q \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$ , such that  $PR : RQ = 1 : 2$ . (3 marks)

*Question 9 continued...*

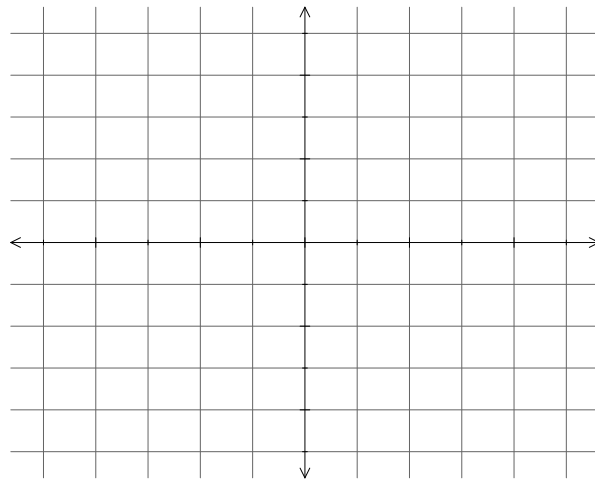
(c) Find an equation of the plane through  $Q \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$  and perpendicular to  $l_1$ , in the form  $r \cdot \vec{n} = \rho$ .

(2 marks)

Draw sketches of the following loci on the Argand diagrams provided.

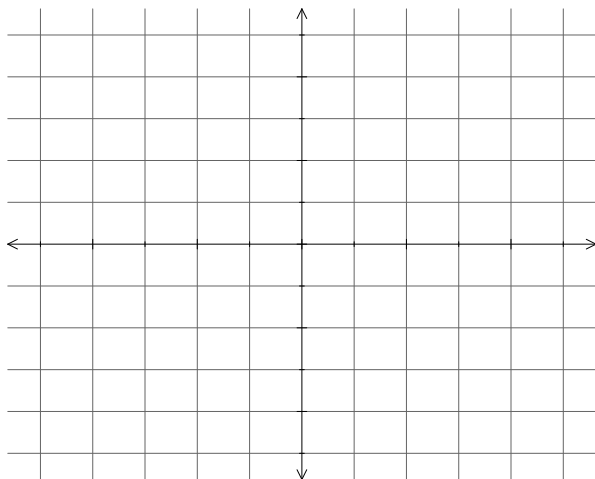
(a)  $\{z : |z| \geq 1 \text{ and } 2 \operatorname{Re}(z) < \operatorname{Im}(z)\}$

(3 marks)



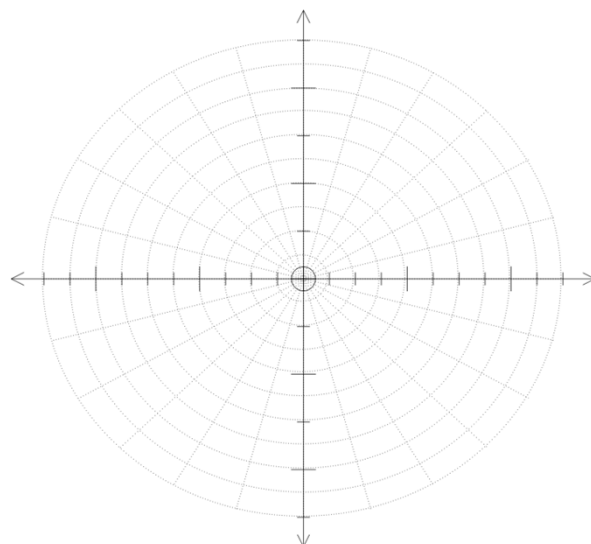
(b)  $\{z : |z + 2 - 2i| \leq |z + 4i|\}$

(3 marks)



(c)  $\left\{z : \frac{\pi}{6} \leq \operatorname{Arg}(z + 1) \leq \frac{\pi}{2}\right\}$

(3 marks)



**Question 11**

**(7 marks)**

Let  $\alpha = -1 + i$ .

(a) Express  $\alpha$  in polar form.

(1 mark)

(b) Show that  $\alpha$  is a root of the equation  $z^4 + 4 = 0$ .

(3 marks)

(c) Hence or otherwise find a real quadratic factor of the polynomial  $z^4 + 4$

(3 marks)

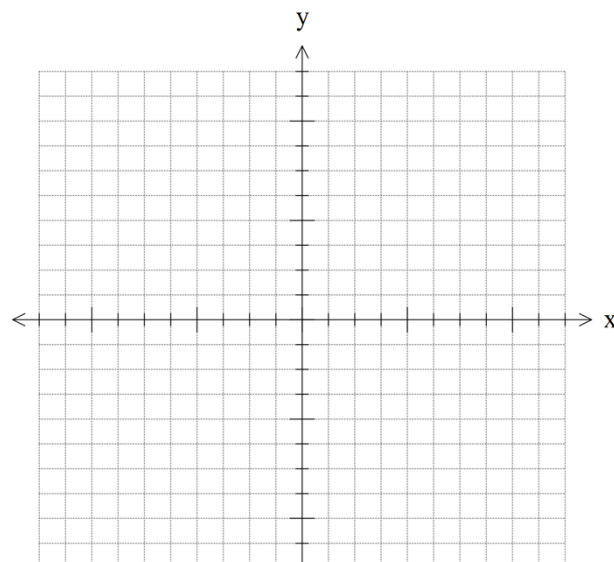
**Question 12**

**(9 marks)**

(a) Sketch the graph of  $f(x)$  described by:

$$f(x) = \begin{cases} |2x| - 1 & -2 \leq x < 0 \\ 1 - |x - 1| & 0 \leq x \leq 3 \end{cases}$$

(3 marks)



(b) Hence, or otherwise, determine  $\int_{-2}^3 f(x) dx$ . (2 marks)

(c) Use an algebraic approach to solve for  $k$  if  $|1 - 2k| \geq 4|k + 3|$ . (4 marks)



## Question 13

(4 marks)

If  $z = \cos \theta + i \sin \theta$ , show that  $\tan \theta = \frac{z - z^{-1}}{i(z + z^{-1})}$ .

## Question 14

(15 marks)

A particle moves such that its position from O at any time,  $t > 0$  is given by

$$\mathbf{r}(t) = (\cos t + t \sin t) \mathbf{i} + (\sin t - t \cos t) \mathbf{j}$$

where  $\mathbf{i}$  and  $\mathbf{j}$  are unit vectors in the East and North direction respectively.

(a) Find, in terms of  $t$  and in simplest form,

(i) the distance of the particle from O,

(3 marks)

(ii) the speed of the particle.

(3 marks)

(b) Find, correct to 3 decimal places, the first non-zero time when the particle is due East or due West of O.

(2 marks)

**Question 14 continued...**

- (c) (i) Find an expression for  $a.r$  in simplest form and use it to find when  $a.r = 0$ .

(5 marks)

- (ii) Interpret the result from (c) (i) in terms of the motion of the particle.

(2 marks)

## Question 15

(11 marks)

(a) The line  $r = \begin{pmatrix} 4+3\lambda \\ 1-3\lambda \\ 1+\lambda \end{pmatrix}$  is inclined to the plane  $r = \begin{pmatrix} 1 \\ a \\ -2 \end{pmatrix} = 4$  at an angle of  $30^\circ$ .

Find  $a$ .

(4 marks)

(b) Find the vector equation of the plane with Cartesian equation  $2x - y = 3 - z$ . (1 mark)

**Question 15 continued...**

- (c) Does the line  $r = 2i + 3j - k + \lambda(3i - j + k)$  lie on the plane  $r \cdot (i + 6j + 3k) = 17$ ?  
Justify your answer.

(2 marks)

- (d) Determine the equation of the plane which is parallel but 5 units away from the plane

$$r \cdot \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} = 4$$

(4 marks)

**Question 16****(16 marks)**

A particle A is projected with **velocity**  $12\mathbf{i} + (24 - 10t)\mathbf{j}$  from the origin O, where  $t$  seconds is the time after projection. At the same time, particle B is projected from  $80\mathbf{i} + 5\mathbf{j}$  so that its **displacement** is given by  $(80 - 20t)\mathbf{i} + (5 + 22t - 5t^2)\mathbf{j}$ . All distances are given in metres.

(a) Which particle was projected with the greatest initial speed, and what was this speed?

(4 marks)

(b) How far apart are A and B after 1 second?

(5 marks)

**Question 16 continued...**

- (c) Which particle reached the greatest height, and how much higher was this than the other particle? (3 marks)

- (d) Show that the particles collide and state the time and position of the collision. (4 marks)

**Question 17****(5 marks)**

Suppose that a complex number  $w$ , lies on the unit circle, and  $0 \leq \arg(w) \leq \frac{\pi}{2}$ .

Prove that  $2 \arg(w+1) = \arg w$ .

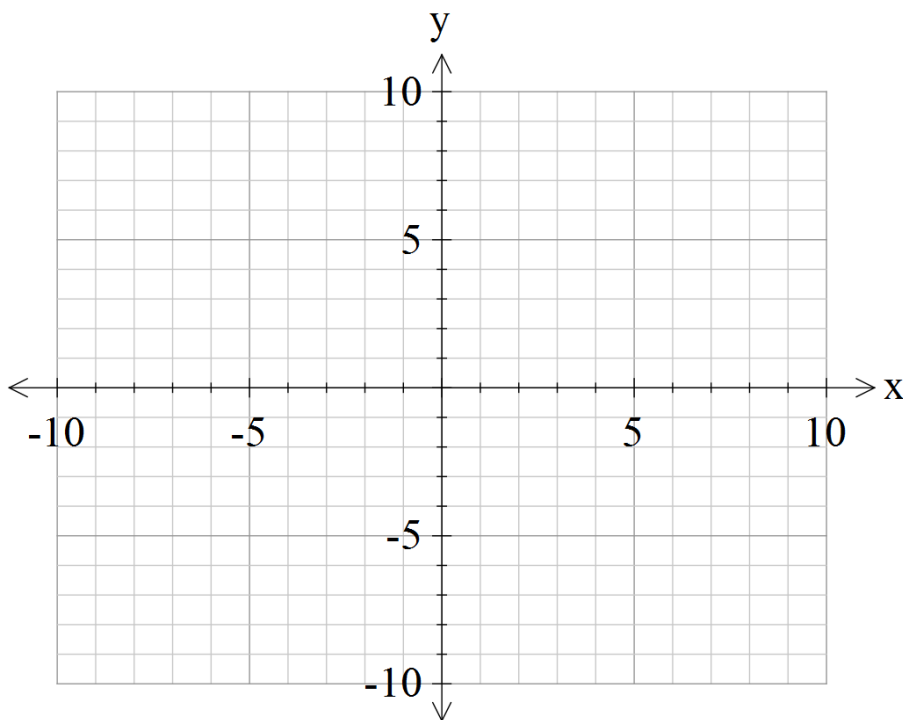


Question 18

(4 marks)

Sketch the graph of the rational function  $f(x)$  given that it has the following properties:

- $f(0) = 0, f'(0) = 0$
- $\lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow 2^+} f(x) \rightarrow \infty$
- $\lim_{x \rightarrow -3^+} f(x) = \lim_{x \rightarrow 2^-} f(x) \rightarrow -\infty$
- $\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow \infty} f(x) = 1$



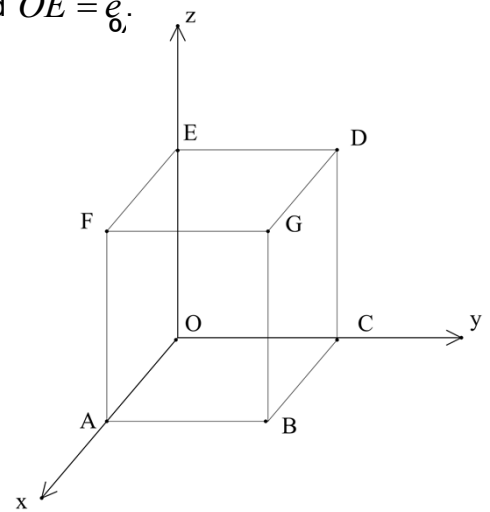
Question 19

(6 marks)

OABCDEFG is a rectangular prism with  $\vec{OA} = \vec{a}$ ,  $\vec{OC} = \vec{c}$  and  $\vec{OE} = \vec{e}$ .

(a) Using the fact that  $|\vec{a} \times \vec{c}| = |\vec{a}| |\vec{c}| \sin \theta$ , show that the area of the base OABC is given by  $\text{Area} = |\vec{a} \times \vec{c}|$ .

(2 marks)



(b) Prove that the volume of the rectangular prism is given by  $(\vec{a} \times \vec{c}) \cdot \vec{e}$ .

(4 marks)

**Additional working space**

Question number: \_\_\_\_\_

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Question number: \_\_\_\_\_